1. Limit Defn of Derivative

(1) $f(x)=x^3$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}=\lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$

$$=\lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h}=\lim_{h\to 0}3x^2+3xh+h^2$$

$$=3x^2$$

(2) $g(x)=1/x^2$

$$g'(x)=\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}=\lim_{h\to 0}\frac{x^2-(x+h)^2}{x^2(x+h)^2h}=\lim_{h\to 0}\frac{-2xh-h^2}{x^2(x+h)^2h}$$

$$=\lim_{h\to 0}\frac{-2x-h}{x^2(x+h)^2}=\frac{-2x}{x^4}$$

$$=-\frac{2}{x^3}$$

(3) $h(x)=\frac{1}{x}+x^2+3$

$$h'(x)=\lim_{h\to 0}\frac{\frac{1}{x+h}+(x+h)^2+3-\frac{1}{x}-x^2-3}{h}=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}+2xh+h^2}{h}$$

$$=\lim_{h\to 0}\frac{(x-(x+h)+2xh(x)(x+h)+h^2(x)(x+h))}{h(x)(x+h)}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(-1+2x\cdot x\cdot(x+h)+h(x)(x+h))}{\cancel{h}(x)(x+h)}$$

$$=\frac{-1+2x^3}{x^2}$$

(4) $f(x)=(5x+2)(3x-1)=15x^2+x-2$

$$f'(x)=\lim_{h\to 0}\frac{(5(x+h)+2)(3(x+h)-1)-(5x+2)(3x-1)}{h}=\lim_{h\to 0}\frac{15(x+h)^2+(x+h)-2-(15x^2+x-2)}{h}$$

$$=\lim_{h\to 0}\frac{15x^2+30xh+15h^2+x+h-2-15x^2-x+2}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(30x+15h+1)}{\cancel{h}}=30x+1$$

2. Derivative Rules

(1) $f(x) = \dfrac{\sqrt{x^2 - x}}{x + x^{-1}}$

$$f'(x) = \frac{\frac{1}{2\sqrt{x^2-x}} \cdot (2x-1) \cdot (x+x^{-1}) - (1 - \frac{1}{x^2})\sqrt{x^2-x}}{(x+x^{-1})^2} =$$

(2) $h(x) = (3\csc x + 1)^{34}$

$$h'(x) = 34(3\csc x + 1)^{33} \cdot (3(-\csc x \cot x)$$

$$= -102 \csc x \cot x (3\csc x + 1)^{33}$$

(3) $g(x) = (3x^2 + x + 1)^{10}(5x+1)^{62}$

$$g'(x) = 10(3x^2+x+1)^9 \cdot (6x+1) \cdot (5x+1)^{62} + (3x^2+x+1)^{10} \cdot 62(5x+1)^{61} \cdot 5$$

$$= 10(3x^2+x+1)^9(6x+1)(5x+1)^{62} + 310(3x^2+x+1)^{10}(5x+1)^{61}$$

(4) $f(x) = 4\sqrt[7]{\sin x \tan x}$

$$f'(x) = 4 \cdot \frac{1}{7}(\sin x \tan x)^{-6/7} \cdot (\cos x \tan x + \sin x \sec^2 x)$$

$$= \frac{4}{7}(\sin x \tan x)^{-6/7}(\cos x \tan x + \sin x \sec^2 x)$$

3. Implicit Differentiation

(1) $\dfrac{y}{x} = xy^4 + 1$

$$\frac{d}{dx}\left(\frac{y}{x}\right) = \frac{d}{dx}(xy^4 + 1)$$

$$\frac{\frac{dy}{dx} \cdot x - y}{x^2} = y^4 + 4xy^3\frac{dy}{dx}$$

$$\frac{dy}{dx} \cdot x - y = x^2y^4 + 4x^3y^3\frac{dy}{dx}$$

$$\frac{dy}{dx} \cdot x - 4x^3y^3\frac{dy}{dx} = x^2y^4 + y$$

$$\frac{dy}{dx}(x - 4x^3y^3) = x^2y^4 + y$$

$$\boxed{\frac{dy}{dx} = \frac{x^2y^4 + y}{x - 4x^3y^3}}$$

$$\frac{d}{dy}\left(\frac{y}{x}\right) = \frac{d}{dy}(xy^4+1)$$

$$\frac{x - y\frac{dx}{dy}}{x^2} = y^4\frac{dx}{dy} + 4xy^3$$

$$x - y\frac{dx}{dy} = x^2y^4\frac{dx}{dy} + 4x^3y^3$$

$$x - 4x^3y^3 = y\frac{dx}{dy} + x^2y^4\frac{dx}{dy}$$

$$\frac{x + 4x^3y^3}{x^2y^4 + y} = \frac{dx}{dy} = \frac{1}{(dy/dx)}$$

(2) $s + \sin t = st$

$$\frac{d}{dt}(s + \sin t) = \frac{d}{dt}(st)$$

$$\frac{ds}{dt} + \cos t = t\frac{ds}{dt} + s$$

$$\frac{ds}{dt} - t\frac{ds}{dt} = s - \cos t$$

$$\frac{ds}{dt}(1-t) = s - \cos t$$

$$\frac{ds}{dt} = \frac{s - \cos t}{1 - t}$$

(3) $\cos(x\tan y) = y^2$

$$\frac{d}{dx}(\cos(x\tan y)) = \frac{d}{dx}y^2$$

$$-\sin(x\tan y)\cdot\left(\tan y + x\sec^2 y\frac{dy}{dx}\right) = 2y\frac{dy}{dx}$$

$$-\sin(x\tan y)\tan y - \sin(x\tan y)\cdot x\sec^2 y\frac{dy}{dx} = 2y\frac{dy}{dx}$$

$$-\sin(x\tan y)\tan y = (\sin(x\tan y)\cdot x\cdot\sec^2 y + 2y)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-\sin(x\tan y)\tan y}{\sin(x\tan y)\cdot x\cdot\sec^2 y + 2y}$$

Slope of tangent line at $(0,1)$: $\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{-\sin(0\cdot\tan 1)\tan 1}{\sin(0\cdot\tan 1)\cdot 0\cdot\sec^2 y + 2\cdot 1} = 0$

Equation: $y - 1 = 0$ OR $y = 1$

4. Intervals of Increase and Decrease

(1) $f(x) = \frac{10x}{x^2+x+1} - 3$

$f'(x) = \frac{10(x^2+x+1) - (10x)(2x+1)}{(x^2+x+1)^2}$

$= \frac{10x^2+10x+10-20x^2-10x}{(x^2+x+1)^2} = \frac{-10x^2+10}{(x^2+x+1)^2}$

Set $f'(x)=0$: $\frac{-10x^2+10}{(x^2+x+1)^2} = 0$

$-10x^2+10 = 0$

$x^2 = 1$

$x = \pm 1$

$f(x)$ inc on: $[-1,1]$

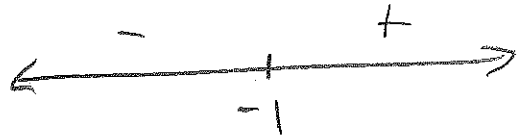
$f(x)$ dec on: $[-\infty,-1] \cup [1,\infty)$

(2) $g(x) = x^2+2x+5$

$g'(x) = 2x+2$

$g'(x)=0$: $2x+2=0$

$x = -1$



$g(x)$ inc on: $[-1,\infty)$

$g(x)$ dec on: $(-\infty,-1]$

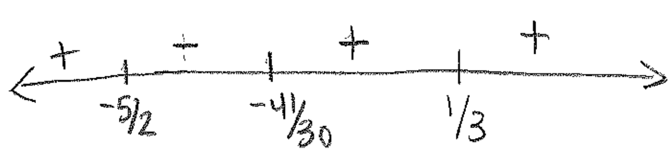
(3) $h(x) = (2x+5)^2(3x-1)^3$

$h'(x) = 2(2x+5)\cdot 2(3x-1)^3 + (2x+5)^2 \cdot 3(3x-1)^2 \cdot 3$

$= (2x+5)(3x-1)^2(4(3x-1)+(2x+5)\cdot 9)$

$= (2x+5)(3x-1)^2(30x+41)$

$h'(x)=0$: $x = -\frac{5}{2}, \frac{1}{3}, -\frac{41}{30}$



$h(x)$ inc on $(-\infty, -5/2] \cup [-41/30, \infty)$

dec on $[-5/2, -41/30]$

5. The Mean Value Thm

(1) (a) $\frac{S(12)-S(0)}{12} = \frac{200(5-\frac{9}{2+12})-200(5-\frac{9}{2})}{12}$

$= 64.28571$

(b) Mean Value Thm

$S'(t) = -1800 \cdot (-1)(2+t)^{-2}$

$= \frac{1800}{(2+t)^2}$

Set $S'(t) = 64.28571$

$\frac{1800}{(2+t)^2} = 64.28571$

$28 = t^2+4t+4$

$0 = t^2+4t-24$

$S = \frac{-4 \pm \sqrt{16+92}}{2} = \frac{-4 \pm \sqrt{108}}{2} =$

negative S doesn't make sense, so only take pos ~~t~~.

$S = 3.196$

so in April

6. Proofs

(1) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{d}{dx}\left(f(x)g(x)^{-1}\right) = f'(x)g(x)^{-1} + f(x)(-1)g(x)^{-2}\cdot g'(x)$

$$= \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{g(x)^2}$$

$$= \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

(2) $\frac{d}{dx}\left(\frac{1}{f(x)}\right) = \frac{d}{dx}(f(x))^{-1} = -1(f(x))^{-2}\cdot f'(x)$

$$= \frac{-f'(x)}{f(x)^2}$$

(3) $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{d}{dx}(\cos x)^{-1} = -1(\cos x)^{-2}\cdot(-\sin x)$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}$$

$$= \tan x \cdot \sec x$$